International Journal of Civil Engineering

Structural Engineering

A simple solution for prediction of steel fiber reinforcd concrete behavior under flexure

R. Ahmadi[1], **P. Ghoddousi**[2,*], **M.Sharifi**[3]

Received: December 2009, Revised: October 2011,Accepted: January 2012

Abstract

The main objective of this study is to drive a simple solution for prediction of steel fiber reinforced concrete (SFRC) behavior under four point bending test (FPBT). In this model all the force components at the beam section (before and after cracking) are formulated by applying these assumptions: a bilinear elastic-perfectly plastic stress-strain response for concrete behavior in compression, a linear response for the un-cracked tension region in a concrete constitutive model, and an exponential relationship for stress-crack opening in the crack region which requires two parameters.Then the moment capacity of the critical cracked section is calculated by applying these assumptions and satisfying equilibrium lawat critical cracked section. After that, parametric studies have been done on the behavior of SFRC to assess the sensitively of model. Finally the proposed model has been validated with some existing experimental tests.The result shows that the proposed solution is able to estimate the behavior of SFRC under FPBT with simplicity and proper accuracy.

Keywords: Steel fiber reinforced Concrete, Flexure, Stress-crack opening, FourPoint Bending Test (FPBT).

---

1. Introduction

Concrete is a relative brittle material and the mechanical behavior of concrete structures is critically influenced by crack propagation. Due to the low fracture toughness of cement-based materials, tensile cracks occur easily due to applied stress, restraint, or environmental conditions. Reinforcement of cementitious materials with short randomly distributed fibers has been successfully used in recent years, primarily for tunnel linings, industrial floor slabs and similar applications [1-5].

Nowadays the question of how to evaluate the flexural toughness and express it as a useful parameter for design purposes is still under debate [6]. A number of test methods have been proposed to evaluate SFRC toughness, but all have significant problems associated with either the variability of the results or their application in structural design calculations [7,8]. Furthermore, in spite of some attempts made at modeling the flexural behavior of steel fiber reinforced concrete, there is not any model that to be able to fully explain what actually happens at the critical cracked section in terms of the fiber-matrix interactions. Consequently, a current limitation on the structural use of the material is a distinct lack of accepted design methods, which has resulted in a lack of confidence in its use as a permanent structural material.

Available methods for modeling the flexural post-crack behavior of SFRC are based on the equilibrium of forces at the cracked section. The evaluation of tensile strength at the crack area which is supported by the fibers bridging is the main point in these methods. Some methods utilizea stress-crack opening models found from direct uniaxial tensile tests or existing relations [9-11]. The main concern is that there is not any accepted uniaxial tensile test method among researchers yet [12] and also using the relations needs iterative solution to determine the required parameters. Soranakom, and Mobasher [13] present the innovative closed form equations for generating moment–curvature response of two classes of fiber reinforced concrete beam; strain hardening and strain softening material.

The other methods implement the average response of the load transmitted by the fibers through the cracked region from pull-out tests of single fibers [14-16]. Some problems arise with this approach, namely the number and position of the fibers bridging in the crack region is not precise and the load

---

* *Corresponding Author: ghoddousi@iust.ac.ir*

1 PHD of Structure, Assistance Professor of BHRC (Building and House Research Centre), Tehran, Iran

2 PHD of Structure, Assistance Professor of Iranian University of Science Technology, Tehran, Iran

3 PHD Candidate of Iranian University of Science Technology, Tehran, Iran

supported by each fiber depends on the crack opening displacement, the orientation of the fiber and the embedment length.

The main objective of this study is to drive a simple solution for prediction of steel fiber reinforced concrete (SFRC) behavior under four point bending test (FPBT). In this study the concrete constitutive model has been supposed as a bilinear elastic-perfectly plastic stress-strain response in compression and a linear response up to tensile strength for tension region. An exponential relation has been assumed for stress-crack opening in crack section. The assumed relation needs two parameters and can be formulated easily. The moment capacity is calculated by applying these assumptions and satisfying equilibrium law in critical cracked section. After that, parametric studies have been done on the behavior of SFRC. Finally proposed model have been validated with some existing experimental test.

2. Model concepts

An idealized representation of strain and stress block diagram at the critical cracked section of a steel fiber reinforced concrete beam under flexural loading is shown in Fig. 1.Considering this figure, the stresses and resultant forces that develop at the critical section can be represented by three separate zones: (1) a compression zone; (2) an un-cracked tension zone; and (3) a cracked tension zone that shows the aggregate interlock and fibers bridging role which the fibers are partially pulled out from the matrix. In this way the flexural capacity of the critical section is the function of the following principal parameters:

- The concrete compressive stress-strain relationship
- The concrete tensile stress-strain relationship
- The concrete crack-opening relationship
- The strain profile and related neutral axis location of uncracked and cracked section
- Crack width profile associated with mid-span deflection.

To predict the shape and magnitude of the stress-block diagram for a given beam deflection, it requires to specify the relation of aforementioned parameters. Hence the flexural moment capacity of the beam for a given beam deflection can be computed when the internal force equilibrium of the section is satisfied.

2.1. Concrete constitutive model relation

An idealized constitutive model has been assumed for concrete in this study as shown in Fig. 2 in order to make simplicity in equations. In this model a bilinear elastic-perfectly plastic stress-strain response has been assumed in compression where the linear portion of response terminates at yield point ($\alpha f_c'$, ε_1) and remains constant at compressive yield stress until the ultimate compressive strain ε_{cu}. The tension response up to tensile strength is described by a linear response with an elastic range defined by E_C.

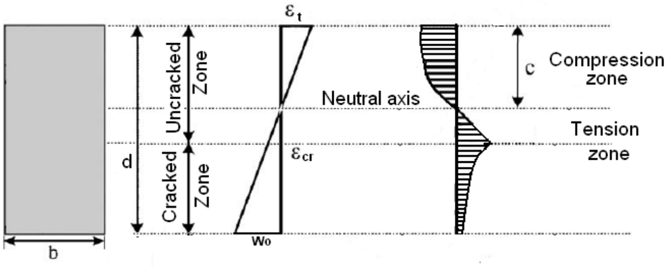


Fig. 1. Strain and stress distribution on crack section

2.2. Crack opening relation

In this study the assumed crack-opening relation is an exponential function as:

$$F_f = f_t\, e^{\beta w} \qquad \beta < 0 \tag{1}$$

Where w is crack opening width and f_t and β are two parameters of model which are tensile strength (f_t) and a coefficient that represents the intensity of descending part of relation respectively. The curve of F_f/f_t in function of β is shown in Fig. 3-a.

This model can describe the tension softening regime of concretes with proper variety. A comparison of supposed model with bilinear model used in Zhang and Stang [7] has been shown in Fig. 3-b.

2.3. Crack Tip opening Width relation

In order to obtain the profile of the curve relating the crack tip opening (w_o) to mid span deflection (δ) in a flexural test it is necessary to make some assumptions. The elastic deflections of the specimen in standard FPBT are of the order of hundredths of a millimeter whereas the desirable region of the load versus deflection diagram for calculating the toughness indices lies in a range of deflections 10 to a 100 times greater than those at first crack i.e. up to 2 mm [17,18]. This implies that rigid body motion of the two broken halves of the specimen is by far the dominant mechanism. So the failure mode commonly observed characterized by a main failure crack at mid-span and the cracked portion at central location acts as a plastic hinge.

Armelin and Banthia [11] proposed a simple solution assuming the axial compressive strain at the top-most fiber of the specimen at mid-span (ε_t) as a function of the rotation

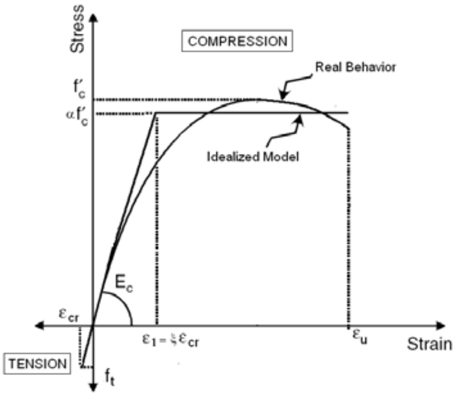
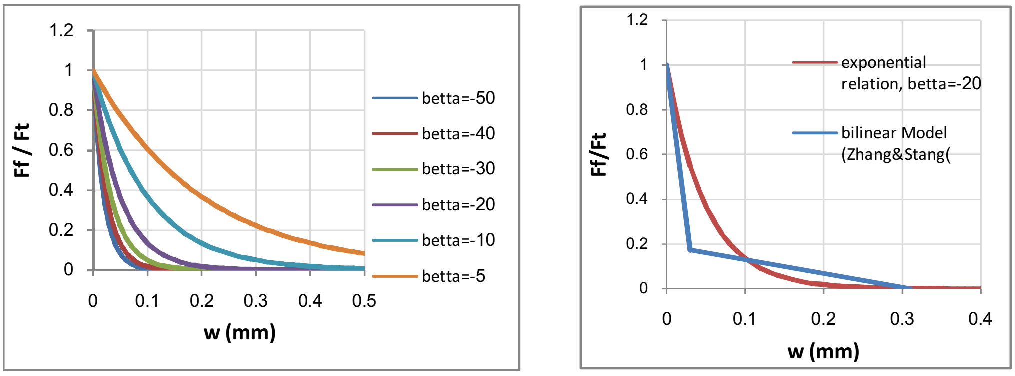


Fig. 2. Idealized constitutive model for concrete



a -The Curve of F_f in function of β

b-Camparision of Model

Fig. 3. The curve of F_f in function of β

angle. The total axial shortening (Δ_t) can be computed from Fig. 4 as:

$$\Delta_t=\int_0^L \varepsilon_x\, dx=2/3\ \varepsilon_t L \quad (2)$$

The deflection at central position is obtained from the slope of the beam as follows:

$$\delta=\theta\, L/2 \quad (3)$$

The linear crack profile is reasonably assumed here in the bending region. The crack width opening displacement (w_o) at the bottom fiber is written as:

$$w_o=2[\theta.(d\text{-}c)\ ] \quad (4)$$

Considering former relation, the stress and force at critical section can be created in three stages as shown in Fig. 5. In order to make simplicity in relations of moment capacity at the critical section, the material parameters are defined in combination of two inherent material parameters including the tensile strain ε_{cr} crand tensile strengthf_t (Fig.2) in addition to module of elasticity (E):

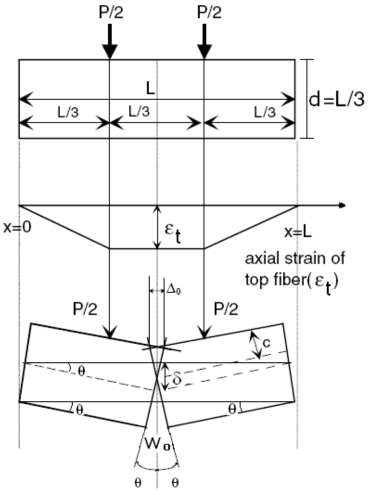
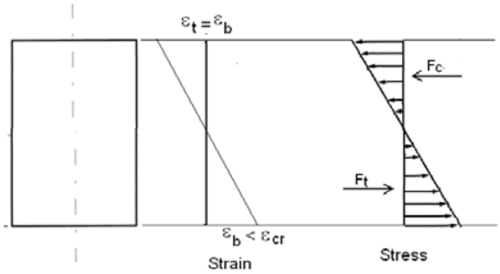
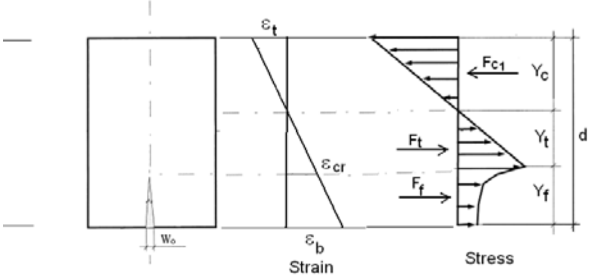


Fig. 4. Failure mode of FRC beam in FPBT

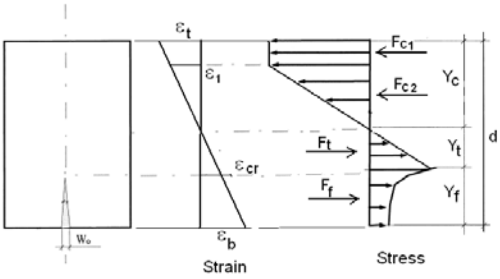
Compressive strain; $\varepsilon_t=\gamma\varepsilon_{cr}$ (5)
Crack strain; $\varepsilon_{cr}=f_t/E$ (6)
Compressive depth; $Y_c=kd \quad k<0.5$ (7)
Characteristic parameters; $\xi=\varepsilon_1/\varepsilon_{cr}=(\alpha f_c')/f_t$ (8)
Uncracked tension depth; $Y_T=kd/\gamma$ (9)
Cracked tension depth; $Y_F=d(1\text{-}k\ (1+\gamma)/\gamma)$ (10)
Where; γ, k are coefficient.



a-Stage 1, before cracking



b-Stage 2, after cracking



c-Stage 3, after cracking

Fig. 5. Stress and force in section

Relations for stage1 ($\varepsilon_b \leq \varepsilon_{cr}$):

The normalized compressive and tensile forces of uncracked section with unit width are obtained as following equation:

Compressive and Tensile Force; $F_c = F_t = \frac{1}{2} E\varepsilon_b \frac{d}{2}$ (11)

Moment Capacity; $M=2F_c \times d/3$ (12)

When the tensile stress at the bottom fiber reaches to tensile strength, one obtains;

Compressive and Tensile Force at cracking state;

$(F_c)_{cr}=(F_t)_{cr}=1/2\ (E\varepsilon_{cr})\ d/2=1/4\ (f_t d)$ (13)

Moment Capacityat cracking state; $M_{cr}=1/6(f_t\, d^2)$ (14)

Relations for stage2, ($\varepsilon_b > \varepsilon_{cr}$,$\varepsilon_t < \varepsilon_o$):

The stage 2 begins while the tensile stress at the bottom fiber of the beam reaches to its strength. In this case, the normalized concrete forces in compression and tension area are calculated with utilization of all normalized parameters defined in Eq. 5 to 10 as following:

Normalized Compressive Force; $F_c=1/2(E\gamma\varepsilon_{cr}Y_c)=1/2\gamma f_t \times kd$ (15)

Normalized Tensile Force; $F_T = \frac{1}{2} E\varepsilon_{cr} Y_T = \frac{1}{2} f_t \times \frac{kd}{\gamma}$ (16)

The force of cracked region can be calculated by carrying out the integral of F_f over the cracked depth (Y_f in Fig. 5-b);

$$F_F = \int_0^{Y_f} f_F dy = \int_0^{Y_f} f_t e^{\beta w} dy = \frac{f_t}{\beta\theta}\left[e^{\beta Y_f \theta} - 1\right] \quad (17)$$

Now considering the equilibrium law at cracked section one obtains:

$$F_c=F_T+F_F \quad (18)$$

$$\frac{1}{2}\gamma kd = \frac{1}{2}\frac{kd}{\gamma} + \frac{1}{\beta\theta}\left[e^{\beta d\left(1-k\frac{1+\gamma}{\gamma}\right)\theta} - 1\right] \quad (19)$$

The Eq. 19 will be used to determine the neutral axis position by changing the depth of compression zone in specified compression strain.

The moment capacity of section can be obtained with the Eq. 20 after meeting the equilibrium law in Eq. 19.

$$M=M_{Fc}+M_{Ft}+M_{Ff} \quad (20)$$

Where M_{Fc}, M_{Ft} and M_{Ff} are the moments derived by stress in compression zone, in tension zone and fiber force in cracked zone respectively about the neutral axis and calculated as;

$$M_{F_C} = F_c \times \frac{2}{3} kd \quad (21)$$

$$M_{F_T} = F_T \times \frac{2}{3}\frac{kd}{\gamma} \quad (22)$$

$$M_{F_F} = F_F \times (\overline{Y}_F + \frac{kd}{\gamma}) \quad (23)$$

$\overline{Y}_F$ is the center of fiber force relative to the lowest uncraked fiber in tension zone, i.e. where the crack width (w) is zero. The $\overline{Y}_F$ is calculated from Eq. 24.

$$\overline{Y}_F = \frac{\int_0^{Y_f} f_t e^{\beta w} \times Y \times dy}{\int_0^{Y_f} f_t e^{\beta w} dy} = \frac{\int_0^{Y_f} f_t e^{\beta w} \times Y \times dy}{F_F} \quad (24)$$

$$\overline{Y}_F = \left[\frac{Y_f e^{\beta Y_f \theta}}{\beta\theta} - \frac{1}{\beta^2\theta^2} e^{\beta Y_f \theta} + \frac{1}{\beta^2\theta^2}\right] \Big/ \left[\frac{1}{\beta\theta}\left(e^{\beta Y_f \theta} - 1\right)\right] \quad (25)$$

Relations for stage 3, ($\varepsilon_b > \varepsilon_{cr}$,$\varepsilon_{tb} > \varepsilon_o$):

While the compressive stress of concrete gets to perfectly plastic region, it is necessary to rewrite the Eq.15, Eq. 19 Eq.21. Ordinary fibrous concrete never experiences this stage; nonetheless the Eq.15, Eq. 19 Eq.21 will substitute as following:

Normalized Compressive Force; $F_C=F_{C1}+F_{C2}$ (26)

$$M_{F_C} = F_{c1}\frac{1}{2}\left(kd + \xi\frac{kd}{\gamma}\right) + F_{c2}\left(\frac{2}{3}\xi\frac{kd}{\gamma}\right) \quad (27)$$

$$kd\alpha\xi\left(1 + \frac{\xi}{\gamma}\right) = \frac{1}{2} \times \frac{kd}{\gamma} + \frac{1}{\beta\theta}\left[e^{\beta Y_f \theta} - 1\right] \quad (28)$$

Where;

$$F_{c1} = \alpha f'_c \times \left(kd - \xi\frac{kd}{\gamma}\right) = kd \times \alpha f'_c(\left(1 - \frac{\xi}{\gamma}\right) \quad (29)$$

$$F_{c2} = \frac{1}{2}\alpha f'_c \times \xi\frac{kd}{\gamma} \quad (30)$$

Now the load-deflection curve can be generated by using the algorithm shown in Fig. 6.

This procedure is simple and can be performed with common mathematical program such as spread sheet in Excel.

Finally it should be noted that this model and its relations can be modified for three point bending test (TPBT) proportionally.

3. Parametric Study of Material Properties

Two sets of parametric studies were conducted. Fig. 7 presents the parametric study ofFRC materials with different crack-opening relations for a specified concrete with f'_c=45 Mpa , f_t=4.5 Mpa and E=33Gpa.

Another parametric study has been done for three sets of FRC material with the same crack-opening relation and different model parameters in tensile strength and module of elasticity. The result has been shown in Fig.8.

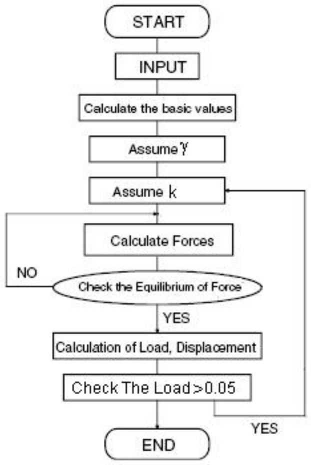


Fig. 6. Algorithm for generating the load-deflection curve

4. Model validation

The proposed model has been validated by using some existing experimental data of theformer researcher. Balagura and Najm [9] performed an experimental investigation on flexural behavior of high-performance fiber-reinforced concrete (FRC) with fiber volume fractions up to 3.75% for 7 mixes. The materials used in this study were cement Types II and V based on ASTM, mortar sand, condensed silica fume, lightweight aggregates, tap water, and admixtures. Three types of fibers were used: 30 x 0.5 mm-diameter hooked steel fibers, 12 mm straight steel fibers (0.2 mm equivalent diameter), and 25x0.4 mm-diameter polypropylene macrofibers. Polypropylene macrofibers are nonmagnetic and have a density of 900 kg/m3 that bond well with concrete. In these mixes the w/c varied from 0.27 to 0.33. The 100 x 100 x 350 mm prisms were tested over a simply supported span of 300 mm. The testing was done under displacement control. The beams were tested at 7, 28, 35, and 64 days. The predicted behavior for the Mix 7 of this study has been compared with experimental result in the Fig. 9.

Another validation has been done with the result of Zhang and Stange [7] studies. They investigated the application of stress crack opening relation in predicting the flexural behavior of FRC. In this study a deformation-controlled of bending tests were carried out on two types of steel fibre concrete beams, straight fibre with circular cross-sections of 0.4 mm and 25 mm in length, and hooked fibre with circular cross-sections of 0.5 mm in diameter and 30 mm in length, here used separately. The size of beam was 420x100 x100mm and the span of bending was 400 mm. Theproposed model and their results in one set data have been shown in Fig. 10.

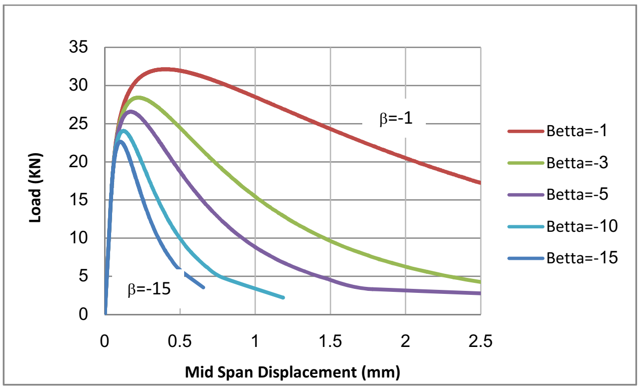


Fig. 7. Load-deflection a FRC material in function of β

5. Conclusion

A simple solution and its relations have been presented for predicting the behavior of FRC material under FPT. Simplicity of solution is due to assumed constitutive model for concrete in compression and tension and proposed crack opening relation. The equations in proposed model have been normalized based on two inherent material parameters: the tensile strain ε_{cr} and tensile strength ft. This model is simple

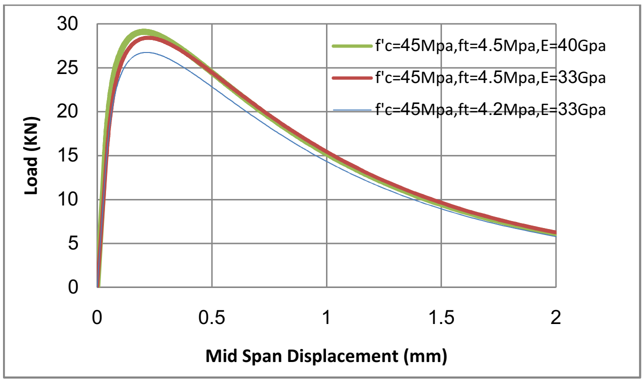


Fig. 8. Load-deflection a FRC material in function of f_t and E

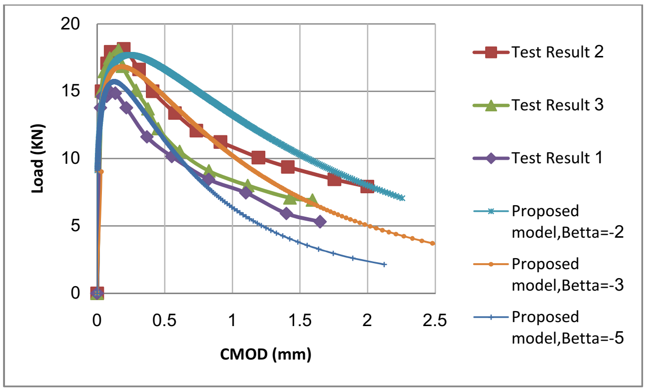


Fig. 10. Comparison of experimental result with proposed model [7]

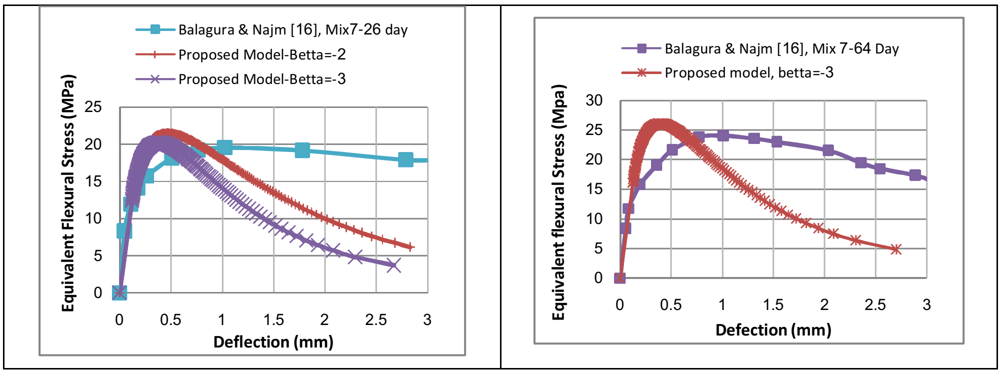


Fig. 9. Comparison of experimental result with proposed model [16]

and can be performed with common mathematical program such as spread sheet in Excel. Parametric study results show that Compressive strength, module of elasticity and also the tensile strength of concrete do not play a major role on post cracking behavior of fibrous concrete, the properties of fiber pullout response, i.e. the crack width opening relation of cracked fibrous concrete mainly affects the post cracking behavior of fibrous concrete.

As seen in above the proposed model has been validated with some existing experimental test. The results have proper agreement with some experimental data and may include some concern in some test. This model can be completed further by using appropriate assumption for crack opening relation.

Acknowledgment: The authors wish to acknowledge theIUSTUniversity(Iranian University of Science and Technology) for their support.

References

[1] Mobasher B, Shah SP. Test parameters for evaluating toughness of glass fiber reinforced concrete panels. ACI Mater J 1989.

[2] Mobasher B, Li CY. Mechanical properties of hybrid cement-based composites. ACI Mater J 1996.

[3] Victor C Li, Large Volume, High-Performance Applications of Fibers in Civil Engineering, High Performance Application of Fiberss,2000.

[4] Dr. Parviz Ghoddousi and ett. All, A Model For Estimating The Aggregate Content For Self Compacting Fiber Reinforced Concrete (SCFRC),International Journal of Civil Engineering , 2010.

[5] M.R. Abdi, Effects of Random fiber Inclusion on Consolidation, Hydraulic Conductivity, Swelling, Shrinkage Limit and Desiccation Cracking of Clays,International Journal of Civil Engineering , 2008.

[6] Henrik Stang, Victor C. Li , 2004, "Classification of Fiber Reinforced Cementitious Material for Structural Application ", 6th RILEM Symposium of FRC,BEFIP 2004.

[7] N. Banthia, J.F. Trottier, Test methods for flexural toughness characterization of fiber reinforced concrete: Some concerns and proposition, ACI Mater J , 1995.

[8] H. Oucief and ett. all, Hybrid fiber reinforced self-compacting concrete: hardened properties,International Journal of Civil Engineering , 2006.

[9] RILEM TC 162-TDF, Test and design Methods for Steel Fiber reinforced Concrete

[10] J. Zhang , H. Stange, Applications of Stress crack Oppening Width Relationship in Predicting the Flexural Behavior of Fiber Reinforced Concrete,Cement and Concrete research, 1997.

[11] H. M. Abdalla and B. L. Karihaloo, A method for constructing the bilinear tension softening diagram of concrete corresponding to its true fracture energy, Magazine of Concrete Research, 56, No. 10, December, 2004.

[12] J. G. M. Van Mier , M. A. A van Vliet, Unixial tensile Test of Determination of Fracture Parameters of Concrete: State of the art, Engineering Fracture mechanics,2001.

[13] Chote Soranakom, Barzin Mobasher, Correlation of tensile and flexural responses of strain softeningand strain hardening cement composites, Cement and Concrete Composite,2008.

[14] Armelin HS, Banthia N (1997) Predicting the flexural post cracking performance of steel fiber reinforced concrete from the pullout of single fibers.ACI Materials Journal, 94.

[15] Byung Hwan Oh et al., Fracture behavior of concrete members reinforced with structural synthetic fibers, Engineering fracture Mechanics,2007.

[16] Luiz Prudencio Jr. et al., Prediction of steel fibre reinforced concrete under flexure from an inferred fibre pull-out response, Materials and Structures , 39:601–610,2006.

[17] ASTM C1018–92 Standard Test Method for Flexural Toughness and First-Crack Strength of Fibre-Reinforced Concrete (Using Beam with Third-Point Loading, ASTM Annual Book of Standards, Vol. 04.02,(1992).

[18] JCI Standards for Test Methods of Fibre Reinforced Concrete. Method of Test for Flexural Strength and FlexuralToughness of Fibre Reinforced Concrete (Standard SF4),(Japan Concrete Institute 1983).

[19] P. Balaguru and H. Najm, "High-Performance Fiber-Reinforced Concrete Mixture Proportions with High Fiber Volume Fractions",ACI Mat. J,2004.