

# Optimal design and operation of irrigation pumping systems using particle swarm optimization algorithm

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**Abstract:** This paper presents a relatively new management model for the optimal design and operation of irrigation water pumping systems. The model makes use of the newly introduced particle swarm optimization algorithm. A two step optimization model is developed and solved with the particle swarm optimization method. The model first carries out an exhaustive enumeration search for all feasible sets of pump combinations able to cope with a given demand curve over the required period. The particle swarm optimization algorithm is then called in to search for optimal operation of each set. Having solved the operation problem of all feasible sets, one can calculate the total cost of operation and depreciation of initial investment for all the sets and the optimal set and the corresponding operating policy is determined. The proposed model is applied to the design and operation of a real-world irrigation pumping system and the results are presented and compared with those of a genetic algorithm. The results indicate that the proposed mode in conjunction with the particle swarm optimization algorithm is a versatile management model for the design and operation of real-world irrigation pumping systems.

**Key words:** pumping stations, design and operation, particle swarm optimization.

## Introduction

The energy required for operating pumping stations in an irrigation district may be significant. Energy costs constitute the largest expenditure for nearly all water utilities worldwide and can consume up to 65% of a water utility annual operating budget. One of the greatest potential areas for energy cost savings lies in the scheduling of pumping operations. The intensive cost of establishing a new pumping station and the ever-increasing cost of energy have caused researchers to pay more attention to the optimal design and operation of pumping stations. Rodin [8] used a GA approach for optimization of a pump scheduling system so as to minimize the cost of pumping over a 24 h period; Ashofteh [1] searched for optimal design of pumping stations and water delivery systems in steady state flow; Boulos

[2] and Moradi-Jalal et al. [7] used a GA in optimization of irrigation pumping stations.

Mathematically, the optimal design and operation of pumping stations is a large-scale nonlinear programming problem because of the size of the problem in terms of the number of the decision variables and nonlinearity of the constraints. The objective in a design and operating of irrigation pumping system problem is to minimize the annual depreciation cost of construction and operations while satisfying system constraints to account for the hydraulics behavior, bounding constraints on decision variables, and other constraints that may reflect the operator preferences or system limitations.

In this paper a new management model for the optimal design and operation of irrigation

water pumping systems is presented. The model makes use of the particle swarm optimization algorithm. A two step optimization model is developed and subsequently solved with the particle swarm optimization method. The model first carries out an exhaustive enumeration search for all feasible sets of pump combinations able to deliver a given demand over the required period. The particle swarm optimization algorithm is then called in to search for optimal operation of each set. Having solved the operation problem of all feasible sets, the total cost of operation and depreciation of initial investment is calculated for all the sets and the optimal set and the corresponding operating policy is determined. The proposed model is applied to the design and operation of a real-world irrigation pumping system and the results are presented and compared with those of a genetic algorithm. The results indicate that the proposed mode in conjunction with the particle swarm optimization algorithm is a versatile management model for the design and operation of real-world irrigation pumping systems.

### Particle Swarm Optimization

The particle swarm optimization method was first introduced by Kennedy and Eberhart [5]. Particle swarm optimization (PSO) is a novel multi agent optimization system (MAOS) inspired by social behavior metaphor. In PSO each agent, call particle, flies in a D-dimensional space S according to the historical experiences of its own and its colleagues. The movement of the particles is stochastic. Each particle keeps track of its coordinates in the problem space. PSO also keeps track of the best solution for all the particles (gbest) achieved so far, as well as the best solution (pbest) achieved so far by

each particle. At the end of a training iteration, PSO changes the velocity of each particle toward its pbest and the current gbest value.

According to the background of PSO and simulation of swarm of bird, Kennedy and Eberhart developed a PSO concept[5]. Suppose that the search space is D-dimensional, then the  $i$ th particle of the swarm can be represented by a D-dimensional vector,  $X_i = (x_{i1}, x_{i2}, \dots, x_{iD})^T$ . The velocity (position change) of this particle, can be represented by another D-dimensional vector,  $V_i = (v_{i1}, v_{i2}, \dots, v_{iD})^T$ . Modification of the agent position (pbest) and its position. This information is analogy of personal experiences of each agent. Moreover, each agent knows the best value so far in the group (gbest) among pbests. This information is analogy of knowledge of how the other agents around them have performed. Namely, each agent tries to modify its positions using the following information:

- The current positions  $X_i$
- The current velocities  $V_i$
- The distance between the current position and pbest
- The distance between the current position and gbest

This modification can be represent by the concept of velocity. Velocity of each agent can be modified by the following equation:

$$v_i^{n+1} = wv_i^n + c_1 rand_1 \times (pbest_i - s_i^n) + c_2 rand_2 \times (gbest - s_i^n) \quad (1)$$

Where  $v_{i,d}^n$  = velocity of agent  $i$  at dimension  $d$ , and iteration  $n$ ;  $w$  = inertia weight;  $c_1, c_2$  = two positive constants, called cognitive and social parameter respectively;  $r_1, r_2$  = random number, uniformly distributed between 0 and 1;  $s_{i,d}^n$  current

position of agent  $i$  at dimension  $d$ , and iteration  $n$ ;  $pbest_{i,d}^n = pbest$  of agent  $i$  at dimension  $d$ , and iteration  $n$ ;  $gbest_{i,d}^n = gbest$  of the group at dimension  $d$ , and iteration  $n$ ;  $i=1,2,\dots,N$ , and  $N$  is the size of the swarm;  $d=1,2,\dots,D$ , and  $D$  is the dimension of search space;  $n=1,2,\dots, iTer_{max}$ , and  $iTer_{max}$  is the maximum of iterations.

The current position (searching point in the solution space) can be modified by the following equation:

$$s_i^{n+1} = s_i^n + v_i^{n+1} \quad (2)$$

Equations (1) and (2) define the initial version of the PSO algorithm. Since there was no actual mechanism for controlling the velocity of a particle, it was necessary to impose a maximum value  $v_{max}$  on it. If the velocity exceeded this threshold, it was equal to  $v_{max}$ . This parameter proved to be crucial, because large values could result in particles moving past good solutions, while small values could result in insufficient of the search space.

The role of the inertia weight,  $w$ , in Equation (1), is considered critical for the PSO,S convergence behavior. The inertia weight is employed to control the impact of the previous history of velocities on the current one. Accordingly, the parameter regulates the tradeoff between the global and local exploration abilities of the swarm. A large inertia weight facilitates global exploration (searching new areas), while a small one tends to facilitate local exploration, i.e., fine-tuning the current search area. A suitable value for the inertia weight usually provides balance between global and local exploration abilities and consequently results in a reduction of the number of iterations required to locate the optimum solution. Initially, the inertia weight was constant.

However, experimental results indicated that it is better to initially set the inertia to a large value, in order to promote global exploration of the search space, and gradually decrease it to get more refined solutions. Thus, Shi and Eberhart [10,11], made a significant improvement in the performance of the PSO with a linearly varying inertia weight over the generations, which linearly vary from  $w_{max}$  at the beginning of the search to  $w_{min}$  at the end. Thus, the following weighting function is usually utilized in Equation (1),

$$w = w_{max} - \frac{(w_{max} - w_{min}) \times n}{iTer_{max}} \quad (3)$$

Where  $w_{max}$ = Initial weight;  $w_{min}$ =Final weight;  $iTer_{max}$ = Maximum iteration number;  $n$ = Current iteration number.

The parameter  $c_1$  and  $c_2$ , in Equation (1), are not critical for PSO,S convergence. However, proper fine-tuning may result in faster convergence and alleviation of local minima [4]. As default values,  $c_1=c_2=2$  were proposed, but experimental results indicate that  $c_1=c_2=0.5$  might provide even better results. Recent work reports that it might be even better to choose a large cognitive parameter,  $c_1$ , than a social parameter,  $c_2$ , but with  $c_1+c_2 \leq 4$ . The parameter  $r_1$  and  $r_2$  are used to maintain the diversity of the population, and they are uniformly distributed in the range (0,1).

## Optimization Model

To implement the optimisation process, an appropriate method may be used to minimize the consumed energy by each generated pumping set, based on the increment time discharge duration curves. The consumed energy,  $E_k$ , is introduced as

$$MinP \cong E_k = \rho g \sum_{j=1}^m \sum_{i=1}^n \frac{Q_{i,j} H_{i,j} (IQ_j)}{e_{i,j} (H_{i,j}, Q_{i,j})} \Delta t_j \quad (4)$$

In which  $E_k$  = total annual consumed energy of  $k$  th pumping set number;  $Q_{ij}$  = discharge from pump  $i$  at time step  $j$ ;  $H_{ij}$  = pumping head of pump  $i$  at time step  $j$ ;  $E_{ij}$  = efficiency of pump  $i$  at time step  $j$ ;  $\Delta t_j$  = time step on the demand-duration curve;  $IQ_j$  = total demand at time step  $j$ ;  $\rho$  = density of water;  $g$  = gravitational acceleration;  $i, j, k$ : subscripts denoting the  $i$ th pump in the  $j$ th division of the demand discharge curve for the  $k$ th pumping set number. Not that pump efficiency is a function of pump discharge and pumping head, which is related to the total discharge at the  $j$ th step.

The objective function, Eq. (4), is constrained by:

$$Q_{i,j} \leq Q_{\max i} \quad (5)$$

$$\sum_{j=1}^m Q_{i,j} = IQ_j \quad (6)$$

$$H_{i,j} \leq H_{\max i} \quad (7)$$

$$H_{i,j} \geq H_{\min i} \quad (8)$$

The net pumping height,  $H_{ij}(IQ_j)$  is also related to the gross pumping head,  $H_{ij}$ , by:

$$H_{i,j} = HS_{i,j} + \sum hf_{i,j} = HS + f \frac{l}{Di} \frac{Vi^2}{2g} \quad (9)$$

Where  $L_i$  = delivery pipe length of pump;  $f$  = friction factor;  $Di$  = delivery pipe diameter of pump;  $HS_{i,j}$  = static head of pump  $i$  duration time step  $j$ ;  $hf_{i,j}$  = friction head loss of pump  $i$  duration time step  $j$ .

The next step is to compute the annual total cost (ATC) for the generated set. An objective function may be formed as

$$\text{Min(ATC)} = \sum_{i=1}^n CRF \times \bar{C}_i + C_E \times E_K \quad (10)$$

$$\bar{C}_i = \left(1 + \frac{r \times CT}{2}\right) \cdot C_i \quad (11)$$

Where  $CRF$  denotes the capital recovery factor defined by:

$$CRF = \frac{r \times (1 + r)^{Tp}}{(1 + r)^{Tp} - 1} \quad (12)$$

$C_E$  and  $C_i$  = unit energy cost and cost of  $i$ th pump, respectively;  $CRF$  = capital recovery factor;  $C_i$  = Equivalent cost of pump  $i$  after construction time;  $CT$  = length of construction time; and  $r$  = interest rate.

For further simplification in the calculation process, it is assumed that the pump efficiency curve is a function of discharge as follows:

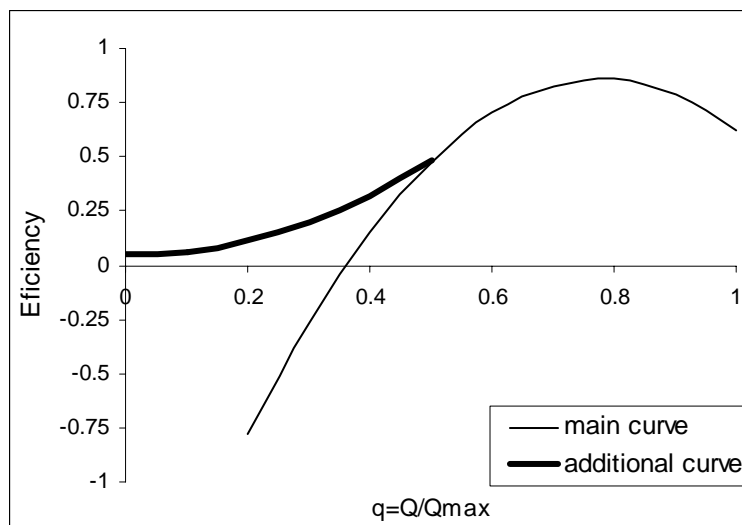
$$e_i(Q_i) = a_i Q_i^2 + b_i Q_i + c_i \quad i = 1, \dots, n \quad (13)$$

Where  $a_i, b_i, c_i$  = coefficients found from the performance curve for the  $i$ th pump by a fitting process.

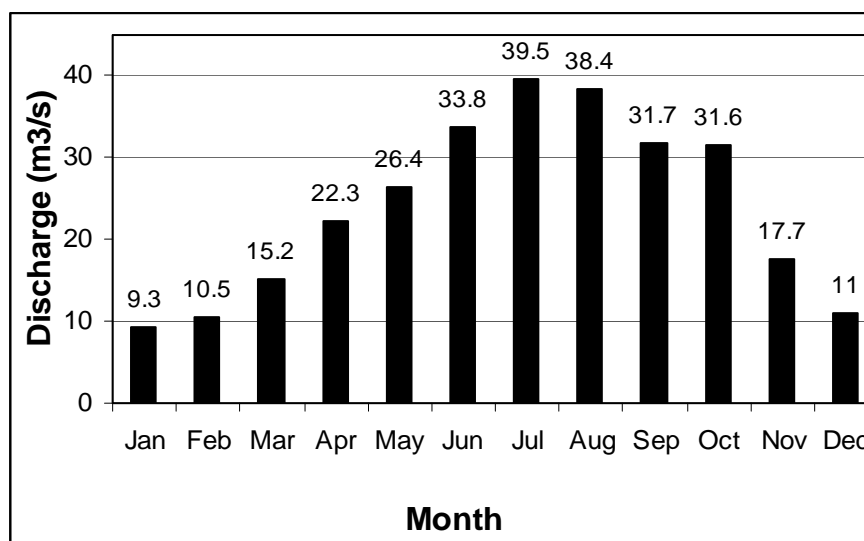
## Model Application

For a case study, the main pumping station of the Farabi Agricultural and Industrial Project is considered. It consists of a 20,000 ha agricultural land, which is located in the Khoozestan province in the southwestern area of Iran. Fig. 2 shows the demand-duration curve and its discretized scheme, which must be pumped by the main pumping station. A problem with one discharge-duration curve, with 12 division representing each month of a year and four different types of pumps for a maximum 10 unit pumps in each pump set, just like the existing design of the Farabi station, was considered to apply.

The optimal pumping set is to be chosen out of four different pump types. The specifications of these pumps including the maximum capacity, optimal capacity, maximum pumping head, length of delivery



**Fig. 1** Efficiency-relative discharge curves for specified pump types



**Fig. 2** Demand diagram histogram in Farabi agricultural and industrial project

pipe and equivalent length of delivery pipe and the initial construction cost are given in Table 1. Fig. 1 shows efficiency-relative discharge curves for specified pump types.

The selected points from the efficiency-relative discharge curves for given pumps are listed in table 2 and are used both in the design example and in the optimized design.

**Table 1** Specification of Pre-selected Pumps

Pump type	Qmax (m3/s)	Qopt (m3/s)	Hmax (m)	Diameter (m)	Leq (m)	Cost (10 <sup>6</sup> rial)
1	7.41	5.70	25	1.35	265.36	224.37
2	2.94	2.26	20	0.9	214.7	89.14
3	2.68	2.06	18	0.8	183.8	82.93
4	1.95	1.50	14	0.7	169.02	59.04

**Table 2** Efficiency-Discharge Relations for Specified Pump Types

Pump type 1		Pump type 2		Pump type 3		Pump type 4	
E(%)	Q(m3/s)	E(%)	Q(m3/s)	E(%)	Q(m3/s)	E(%)	Q(m3/s)
86.00	5.70	86.00	2.26	86.00	2.06	86.00	1.50
81.70	5.13	81.70	2.03	81.70	1.85	81.70	1.35
83.20	6.27	83.20	2.49	83.20	2.27	83.20	1.65
75.20	4.56	75.20	1.810	75.20	1.65	75.20	1.20
79.10	6.84	79.10	2.71	79.10	2.47	79.10	1.80

It is important to note that in the optimization model “relative discharge” which is the ratio of discharge to maximum allowable discharge of each pump,  $q_i = \frac{Q_i}{Q_{\max i}}$ , is selected

as the decision variables to simplify the calculation. Furthermore, the following values  $r=0.06$  per year,  $CT=3$  years, and  $TP=20$  years are used so that a fair comparison could be made between the results of the optimization model and that of engineering design.

In a real case, in order to avoid the problem of division by zero in the calculation of Eq. (4), two different curves were considered for efficiency-relative discharge curves to prevent the reporting of infeasible and incorrect discharge result. Thus, by applying two curves, the main curve,  $e_i = a_i q_i^2 + b_i q_i + c_i$ , and the additional curve, in the program without losing the final optimal results, diversion of the program to unfeasible results

is removed during the calculation process. Table 4 shows the number of pumps and annual total cost for 10 first Optimum sets are while a typical output of discharge for optimum set of pumps is shown in Table 6. Table 5 illustrates the detail of the optimum solution obtained using the proposed PSO model. To illustrate the type of the objective function involved in the model, a 3-D plot of the consumed energy for a system composed of two pump type 1 is illustrated in Fig. 3. Although, this variation seems to have a unimodal nature, the real objective function could exhibit multimodal feature as it is a superposition of some nearly unimodal function with different optimum points. Note that Fig. 3 only illustrates the consumed energy and does not include the construction cost. For further clarification of the model efficiency, the convergence history of the global best particle to the optimum solution is also shown in Fig. 4. This figure show the ability of the PSO algorithm to quickly locate

**Table 3** Coefficients of efficiency-relative discharge curve for specified pump types

Coefficient	Additional curve ( $q < 0.5$ )	Main curve ( $q \geq 0.5$ )
$a_i$	1.840	-4.870
$b_i$	-0.060	7.603
$c_i$	0.050	-2.107

**Table 4** Number of pumps and annual total cost for 10 first optimum set

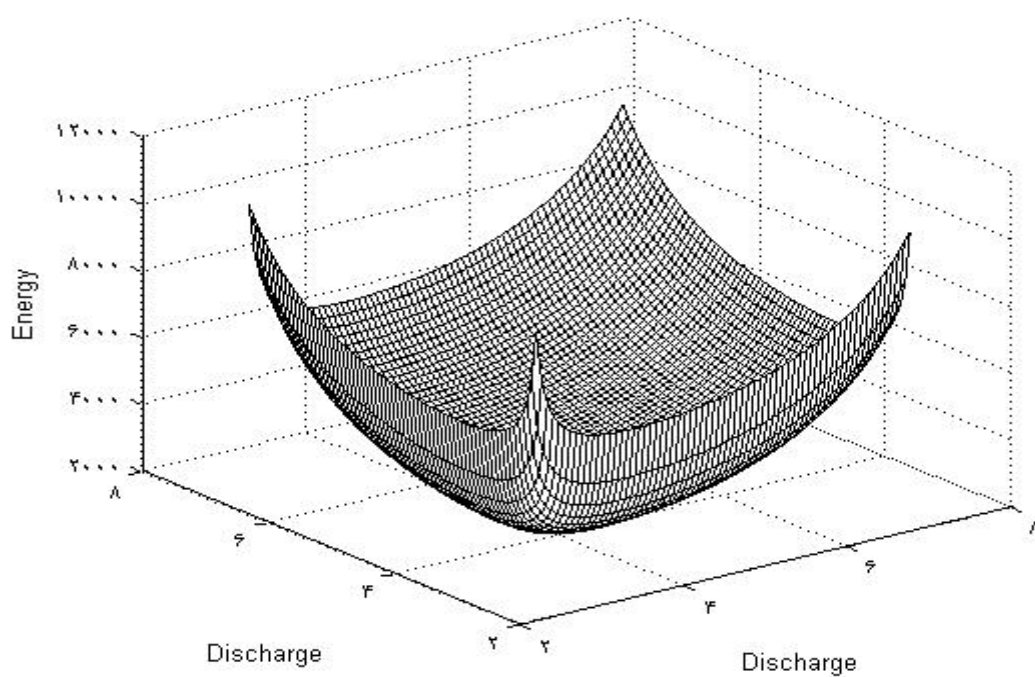
Set No.	Pump Type 1	Pump Type 2	Pump Type 3	Pump Type 4	Total No. of Pumps	Annual total cost ( $10^6$ rial)
1	4	2	1	1	8	193.7
2	4	4	0	0	8	194.7
3	4	2	2	0	8	195.1
4	5	0	0	3	8	196.0
5	3	3	1	3	10	197.3
6	3	3	3	1	10	197.8
7	4	3	0	1	8	198.3
8	3	4	3	0	10	199.8
9	5	0	1	1	7	200.0
10	4	0	4	0	8	200.1

**Table 5** Specifications of the final solution obtained by proposed PSO model

Cost ( $10^6$ rial)	PSO
Annual depreciation cost	115.7
Annual operation cost	78.0
Annual total cost	193.7
Pumping systems	4 - 2 - 1 - 1

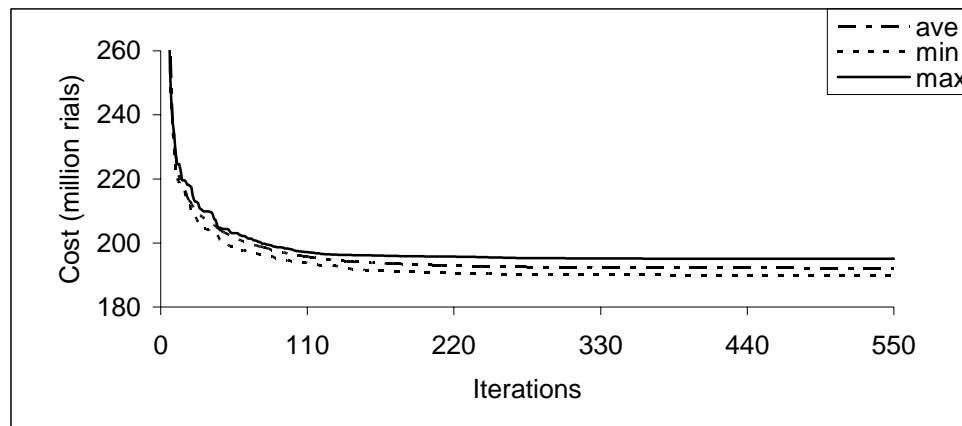
**Table 6** Typical output of discharge for optimum set of pumps

month	demand (m <sup>3</sup> /s)	pump 1 (m <sup>3</sup> /s)	pump 2 (m <sup>3</sup> /s)	pump 3 (m <sup>3</sup> /s)	pump 4 (m <sup>3</sup> /s)	pump 5 (m <sup>3</sup> /s)	pump 6 (m <sup>3</sup> /s)	pump 7 (m <sup>3</sup> /s)	pump 8 (m <sup>3</sup> /s)
Jul	39.5	7.41	6.58	6.45	6.86	2.44	2.30	1.60	1.74
Aug	38.4	6.65	7.41	6.53	7.41	2.39	2.34	1.73	1.70
Jun	33.8	7.41	6.31	6.60	7.24	2.42	2.30	0	1.70
Sep	31.7	6.22	5.86	6.02	5.92	0	2.32	0	1.50
Oct	31.6	5.74	6.38	6.30	6.21	2.21	2.15	0	0
May	26.4	5.37	5.76	5.52	5.62	1.96	2.06	0	1.44
Apr	22.3	4.49	4.66	4.62	4.53	2.12	2.18	1.85	1.25
Nov	17.7	4.96	0	4.68	4.77	2.67	0	1.77	0
Mar	15.2	4.48	0	4.46	4.51	0	2.68	2.00	0
Dec	11.0	0	5.19	4.56	0	1.67	0	1.61	1.24
Feb	10.5	4.87	0	0	0	1.58	0	2.68	0
Jan	9.3	0	0	4.65	4.56	1.67	1.61	0	0



**Fig.3** 3-D plot of the consumed energy for a system composed of 2 pump type 1.





**Fig.4** Convergence history of the Minimum, Maximum and Average solution cost over 20 runs.

the area containing optimal or near optimal solutions.

### Concluding Remarks

In this paper, a new evolutionary algorithm namely particle swarm optimization is used to solve the problem of design and operation of irrigation pumping systems. Mathematically, the optimal design and operation of pumping stations is a large-scale nonlinear programming problem because of the size of the problem in terms of the number and nonlinearity of the decision variables and constraints. The proposed optimization model is a two step model. The model first uses an enumeration scheme to find the set of possible pump combinations able to supply a given demand-curve. In the second step, the PSO model is used to minimize the total cost, consisting of operation and depreciation cost of initial investment, by changing the set and discharge of pump sets based on the provided feasible. For a case study, the main pumping station of the Farabi Agricultural and Industrial Project is solved and the results are

presented.

### References

- [1] Ashofteh, J. (1999). "Optimal design of pumping stations and water delivery systems in steady state flow.", Proc., 2nd Conf. on Hydraulics of Iran, A. Afshar, ed., Tehran, Iran, 306-319.
- [2] Boulos, P. E., Wu, Z. Y., Orr, C. H., and de Schaetzn, W. (2001a). "Using genetic algorithms for water distribution system optimization." Proc., ASCE Environmental resource Congress, (CD-ROM), ASCE, New York, Sec. 1, Chap 472.
- [3] Kennedy, J., and Eberhart, R., "Particle Swarm Optimization", Proceedings of the International Conference on Neural Networks, Perth, Australia, 1995 IEEE, Piscataway, 1995, pp. 1942-1948.
- [4] Kennedy, J., "The behavior of particles" In: Porto VW, Saravanan N, Waagen D and Eiben AE (eds) Evolutionary Programming VII, 1998, pp. 581-590.

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- [5] Konstantinos, E.P., Vrahatis, M.N., "Particle Swarm Optimization method for constrained optimization problems.",UPARIC,GR-26110 Patras,Greece.
- [6] Moradi-Jalal, M., Sergey I.Rodin, Hon.M.ASCE , 2004," Use of Genetic Algorithm in optimization of irrigation pumping station.", Journal of irrigation and drainage engineering, September, 2004,p.p357-365
- [7] Moradi-Jalal,M.,Marino,M.A.,and Afshar,A.(2003)."Optimal design and operation of irrigation pumping station."J.Irrig.Drain.Eng.,129(3),149-154.
- [8] Rodin, S. I., (1998). "Use of genetic algorithm for optimal control of bulk water supply." <<http://stullia.t-k.ru/waterpump/waterpump.htm>> (May 5, 2001).
- [9] Rodin, S. I., and Moradi-Jalal, M. (2002). "Use of genetic algorithm in optimization of irrigation pumping stations, WAPIRRA program." <<http://stullia.t-k.ru/waterpump/waterpump.htm>> (June 10,2002).
- [10] Shi, Y, and Eberhart, RC. "Empirical study of particle swarm optimization", Proceedings IEEE International Congress Evolutionary Computation, Washington , DC., USA,1999, pp. 1945-50.
- [11] Shi, Y, and Eberhart, R, "Parameter selection in Particle Swarm Optimization", In: Porto VW, Saravanan N, Waagen D and Eiben AE (eds) Evolutionary Programming VII, 1998,pp. 611-616.